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Holographic Grand Unification

Yasunori Nomura, David Poland and Brock Tweedie

Department of Physics, University of California, Berkeley, CA 94720
Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720

Abstract

We present a framework for grand unification in which the grand unified symmetry is broken spontaneously by strong gauge dynamics, and yet the physics at the unification scale is described by (weakly coupled) effective field theory. These theories are formulated, through the gauge/gravity correspondence, in truncated 5D warped spacetime with the UV and IR branes setting the Planck and unification scales, respectively. In most of these theories, the Higgs doublets arise as composite states of strong gauge dynamics, corresponding to degrees of freedom localized to the IR brane, and the observed hierarchies of quark and lepton masses and mixings are explained by the wavefunction profiles of these fields in the extra dimension. We present several realistic models in this framework. We focus on one in which the doublet-triplet splitting of the Higgs fields is realized within the dynamical sector by the pseudo-Goldstone mechanism, with the associated global symmetry corresponding to a bulk gauge symmetry in the 5D theory. Alternatively, the light Higgs doublets can arise as a result of dynamics on the IR brane, without being accompanied by their triplet partners. Gauge coupling unification and proton decay can be studied in these models using higher dimensional effective field theory. The framework also sets a stage for further studies of, e.g., proton decay, fermion masses, and supersymmetry breaking.

1 Introduction

Weak scale supersymmetry (SUSY) provides an elegant solution to the naturalness problem of the standard model, by invoking a cancellation between the standard model and its superpartner contributions to the Higgs potential. An interesting consequence of this framework is that the three gauge couplings unify at an extremely high energy of order $M_U \approx 10^{16}$ GeV, if a normalization of the $U(1)_Y$ gauge coupling is adopted that allows the embedding of the standard model gauge group into a larger simple symmetry group: $SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$. This suggests the existence of some unified physics above this energy scale, which in some form utilizes $SU(5)$ or a larger group containing it.

The simplest possibilities for physics above M_U are four dimensional (4D) supersymmetric grand unified theories (GUTs) [1]. In these theories, physics above M_U is described by 4D supersymmetric gauge theories in which the standard model gauge group is embedded into a larger (simple) gauge group. This, however, leads to the problem of doublet-triplet splitting in the Higgs sector, and often leads to too rapid proton decay caused by the exchange of colored triplet Higgsinos [2]. While several solutions to these problems have been proposed within conventional 4D SUSY GUTs [3 – 7], their explicit implementations often require the introduction of a larger multiplet(s) and/or specifically chosen superpotential interactions, especially when one tries to make the models fully realistic. This loses a certain beauty the simplest theory had, especially if one adopts the viewpoint that these theories are "fundamental," arising directly from physics at the gravitational scale, e.g. string theory.

An alternative possibility for physics above M_U is that the unified gauge symmetry is realized in higher dimensional (semi-)classical spacetime [8 – 10]. In this case there is no 4D unified gauge symmetry containing the standard model gauge group as a subgroup — the unified symmetry in higher dimensions is broken locally and explicitly by a symmetry breaking defect. This structure allows a natural splitting between the doublet and triplet components for the Higgs fields, while the successful prediction for gauge coupling unification is recovered by diluting the effects from the defect due to a moderately large extra dimension(s). Dangerous proton decay is suppressed by an R symmetry, arising naturally from the higher dimensional structure of the triplet Higgsino mass matrix. The framework also allows for a simple understanding of the observed structure of fermion masses and mixings, in terms of wavefunction suppressions of the Yukawa couplings arising for bulk quarks and leptons [11, 12].

In this paper we study a framework for physics above M_U in which the standard model gauge group is unified into a simple gauge group in precisely the same sense as in conventional 4D SUSY GUTs, and yet mechanisms and intuitions developed in higher dimensions can be used to address the various issues of unified theories. Let us consider that the standard model gauge group is embedded into a simple unified gauge group, e.g. $SU(5)$, at energies above M_U. We assume that the unified gauge symmetry is broken by strong gauge dynamics associated with another gauge group G, and that this gauge group has a large 't Hooft coupling $\tilde{g}^2\tilde{N}/16\pi^2 \gg 1$, where $\tilde{g}$ and $\tilde{N}$ are the gauge coupling and the number of "colors" for the gauge group G. With these values of the 't Hooft coupling for G, an appropriate (weakly coupled) description of physics is given in higher dimensional warped spacetime (for $\tilde{N} \gg 1$), due to the gauge/gravity correspondence [13]. In the simplest setup where $\tilde{g}$ evolves slowly above the dynamical scale, our theories are formulated in 5D anti-de Sitter (AdS) spacetime truncated by two branes, where the curvature scales on the ultraviolet (UV) and infrared (IR) branes are chosen to be $k \approx (10^{17}-10^{18})$ GeV and $k' \approx (10^{16}-10^{17})$ GeV, respectively. This allows us to construct simple "calculable" unified theories in which the unified gauge symmetry is broken dynamically — physics above M_U is determined simply by specifying parameters in higher dimensional effective field theory.

In this paper we construct realistic unified theories in the framework described above. In general, there are many ways to address the issues of unified theories in our framework. In one example, which we discuss in detail, we use the idea that the Higgs doublets of the minimal supersymmetric standard model (MSSM) are pseudo-Goldstone bosons of a broken global symmetry [6, 14]. Specifically, we assume that the G sector possesses a global $SU(6)$ symmetry, of which an $SU(5)$ $(\times U(1))$ subgroup is gauged. The gauged $SU(5)$ group contains the standard model gauge group as a subgroup. We assume that the dynamics of G breaks the global $SU(6)$ symmetry down to $SU(4) \times SU(2) \times U(1)$ at the scale M_U, which leads to the correct gauge symmetry breaking, $SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$, and ensures that the Higgs doublets remain massless after the symmetry breaking, without being accompanied by their colored triplet partners. The simplest realization of our theory, corresponding to this symmetry structure, is then obtained in 5D truncated warped space in which the bulk $SU(6)$ gauge symmetry is broken to $SU(5) \times U(1)$ and $SU(4) \times SU(2) \times U(1)$ on the UV and IR branes, respectively. Realistic unified theories having this symmetry structure were constructed previously in flat space in Ref. [15], where the symmetry breakings on the two branes are both caused by boundary conditions, and in Ref. [16], where the breakings are by the Higgs mechanism. In our context, we find that the simplest theory is obtained if the breakings on the UV and IR branes are caused by boundary conditions and the Higgs mechanism, respectively. Note that, in the "4D description" of the theory, the Higgs breaking on the IR brane corresponds to dynamical GUT breaking, and the low-energy Higgs doublets are interpreted as composite particles of the dynamical GUT-breaking sector. This theory thus provides a simple explicit realization of the composite pseudo-Goldstone Higgs doublets, in which the origin of the global $SU(6)$ symmetry can be understood as the "flavor" symmetry of the dynamical GUT-breaking sector.

Below the GUT-breaking scale M_U, our theory is reduced to the MSSM (supplemented by small seesaw neutrino masses). The successful unification prediction for the low-energy gauge couplings is preserved as long as the threshold corrections from the dynamical GUT-breaking sector are sufficiently small. Our higher dimensional description of the theory allows us to estimate the size of these corrections, and we find that this can be the case. Dimension five proton decay does not exclude the theory, because of the existence of these threshold corrections. Realistic quark and lepton mass matrices can also be reproduced, where the observed hierarchies in masses and mixings are understood in terms of the wavefunction profiles of the quark and lepton fields. In the 4D description of the theory, these hierarchies arise through mixings between elementary states and composite states of G, which are given by powers of M_U/M_*, where M_* is the fundamental scale of the theory, close to the 4D Planck scale. Unwanted unified mass relations for the first two generation fermions do not arise, because of GUT breaking effects in the G sector.

We also discuss other possible theories in our framework. We show that it allows for the construction of large classes of models, including missing partner type and product group type models. In most of them, the Higgs doublets arise as states localized to the IR brane, corresponding to composite states of the strong G dynamics. A 4D scenario related to these theories was discussed previously in Ref. [17], based on a supersymmetric conformal field theory (CFT), where a possible AdS interpretation was also noted. In all of these theories, our higher dimensional framework allows a straightforward implementation of the mechanism generating the hierarchical fermion masses and mixings, in terms of the wavefunction profiles of matter fields in the extra dimension.

The organization of the paper is as follows. In the next section we describe the basic structure of our theory using the 4D description. We describe how the MSSM arises naturally at low energies in this theory. In section 3 we construct an explicit model in truncated 5D warped space. We show that the model does not suffer from problems of conventional 4D SUSY GUTs, e.g. the doublet-triplet splitting and dimension five proton decay problems, and also that the observed hierarchies in the quark and lepton mass matrices can be understood in terms of the wavefunction profiles of these fields in the extra dimension. In section 4, we discuss other possible theories in our framework, including missing partner type and product group type models. Discussion and conclusions are given in section 5, which include a comment on the possibility of having a theory with $\tilde{g}^2\tilde{N}/16\pi^2 \lesssim 1$.

2 Basic Picture

In this section we describe our theory using the 4D description. Here we focus on the case where the light Higgs doublets of the MSSM arise as pseudo-Goldstone supermultiplets of the GUT scale dynamics. This has the virtue that the success of the theory is essentially guaranteed by its symmetry structure, without relying on specifically chosen matter content or interactions. Other possibilities will be discussed in section 4.

We consider that the standard model gauge group is embedded into a simple gauge group $SU(5)$, which is spontaneously broken at the scale $M_U \approx 10^{16}$ GeV. What is the underlying dynamics of this symmetry breaking? A hint will come from considering how the MSSM arises below the symmetry breaking scale M_U. In particular, considering how the MSSM matter content naturally appears at energies below M_U and why interactions among these particles – the gauge and Yukawa interactions – take the observed form and values will provide a guide to the physics of this symmetry breaking. The suppression of certain operators allowed by standard model gauge invariance, e.g. the ones leading to dangerous dimension five proton decay, may also give hints regarding the structure of this physics.

We focus on the possibility that the unified gauge group, $SU(5)$, is spontaneously broken by dynamics associated with another gauge group G. In this setup, the G sector is charged under $SU(5)$, as it breaks $SU(5)$ dynamically. The setup also allows the existence of other fields – elementary fields – that are singlet under G and charged under $SU(5)$. Suppose now that the theory has a matter content that satisfies $n_{\mathbf{5}^*} - n_{\mathbf{5}} = n_{\mathbf{10}} - n_{\mathbf{10}^*} = 3$ and $n_{\mathbf{r}} - n_{\mathbf{r}^*} = 0$ ($\mathbf{r} \neq \mathbf{5}, \mathbf{10}$), where $n_{\mathbf{r}}$ represents the number of $SU(5)$ multiplets in a complex representation $\mathbf{r}$. The matter content is arbitrary otherwise. (Note that this is not a very strong requirement on the spectrum — with $n_{\mathbf{r}} - n_{\mathbf{r}^*} = 0$ for $\mathbf{r} \neq \mathbf{5}, \mathbf{10}$, the condition $n_{\mathbf{5}^*} - n_{\mathbf{5}} = n_{\mathbf{10}} - n_{\mathbf{10}^*}$ arises automatically as a consequence of anomaly cancellation.) With this assumption, the low energy matter content is expected to be just the three generations of quarks and leptons, no matter what happens associated with the dynamics of the GUT-breaking sector G. In general, the gauge dynamics of G will produce an arbitrary number of split GUT multiplets as composite states, by picking up the effect of GUT breaking. These states can then mix with the elementary states, so that the low energy states are in general mixtures of elementary and composite states and thus a collection of various incomplete $SU(5)$ multiplets. Nevertheless, conservation of chirality guarantees that we always have three generations of quarks and leptons at low energies, although they may not arise simply from three copies of $(\mathbf{5}^* + \mathbf{10})$. Assuming that all the fields vector-like under the standard model gauge group obtain masses of order M_U through nonperturbative effects of G, the matter content below M_U is exactly the three generations of quarks and leptons.

The above argument shows that we can naturally obtain a low-energy chiral matter content that fills complete $SU(5)$ multiplets for chirality reasons (although each component in a multiplet may come from several different $SU(5)$ multiplets at high energies). It also implies that any multiplets that do not fill out a complete $SU(5)$ multiplet must be vector-like. It is interesting that the MSSM has exactly this structure. Unless there is some special reason, however, the vector-like states are all expected to have masses of order M_U from nonperturbative effects of G. What could the special reason be for the Higgs doublets?

The lightness of the Higgs doublets can be understood group theoretically if we identify these states as pseudo-Goldstone bosons of a broken global symmetry [6]. Suppose that the G sector possesses a global $SU(6)$ symmetry, of which an $SU(5)$ ($\times U(1)$) subgroup is gauged and identified as the unified gauge symmetry. We assume that the dynamics of G breaks the global $SU(6)$ symmetry down to $SU(4) \times SU(2) \times U(1)$ at the dynamical scale $\approx M_U$ in such a way that the gauged $SU(5)$ subgroup is broken to the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ (321). This leads to Goldstone chiral supermultiplets, whose quantum numbers under 321 are given by $(\mathbf{3}, \mathbf{2})_{-5/6} + (\mathbf{3}^*, \mathbf{2})_{5/6} + (\mathbf{1}, \mathbf{2})_{1/2} + (\mathbf{1}, \mathbf{2})_{-1/2}$. While the first two of these are absorbed by the broken $SU(5)$ gauge multiplets (the massive XY gauge supermultiplets), the last two are left in the low energy spectrum. Although the global $SU(6)$ symmetry of the G sector is explicitly broken by the gauging of the $SU(5)$ ($\times U(1)$) subgroup, the supersymmetric nonrenormalization theorem guarantees that the mass term for $(\mathbf{1}, \mathbf{2})_{1/2} + (\mathbf{1}, \mathbf{2})_{-1/2}$ is not generated without picking up the effect of supersymmetry breaking, allowing us to identify these states as the two Higgs doublets of the MSSM: $H_u(\mathbf{1}, \mathbf{2})_{1/2}$ and $H_d(\mathbf{1}, \mathbf{2})_{-1/2}$. This provides a complete understanding of the MSSM field content in our framework. The MSSM states – the gauge, matter and Higgs fields – are the only states that could not get a mass of order M_U from G, because they are protected by gauge invariance, chirality, and the (pseudo-)Goldstone mechanism.

Since the two Higgs doublets arise from the dynamical breaking of $SU(6)$, they are composite states of G. Suppose now that the dynamics of G also produces composite states that have the same 321 quantum numbers as the MSSM quarks and leptons, $\mathcal{Q}, \mathcal{U}, \mathcal{D}, \mathcal{L}$ and $\mathcal{E}$. These composite states will then have "Yukawa couplings" with the Higgs fields at M_U, $W \approx \mathcal{Q}\mathcal{U}H_u + \mathcal{Q}\mathcal{D}H_d + \mathcal{L}\mathcal{E}H_d$, where the sizes of the couplings are naturally of order 4π. These couplings, however, disappear at low energies after integrating out all the heavy modes, because the strong G dynamics respects $SU(6)$ and the Higgs doublets are the Goldstone bosons associated with the dynamical breaking of $SU(6)$. Now, suppose that the theory also has several elementary fields that transform as $\mathbf{5}^*$ and $\mathbf{10}$ under $SU(5)$. In this case the low-energy quarks and leptons, Q, U, D, L and E, are in general linear combinations of the elementary and composite states. The Yukawa couplings for these low-energy fields, $W \approx QUH_u + QDH_d + LEH_d$, can then be nonzero because the elementary states do not respect the full $SU(6)$ symmetry. The sizes of the Yukawa couplings are determined by the strengths of the mixings between the elementary and composite states, which are in turn determined by the dimensions of the G-invariant operators that interpolate the composite states. This situation is analogous to the case where the standard model Higgs boson is identified as a pseudo-Goldstone boson of strong gauge dynamics at the TeV scale [18]. By choosing operator dimensions to be larger for lighter generations, we can

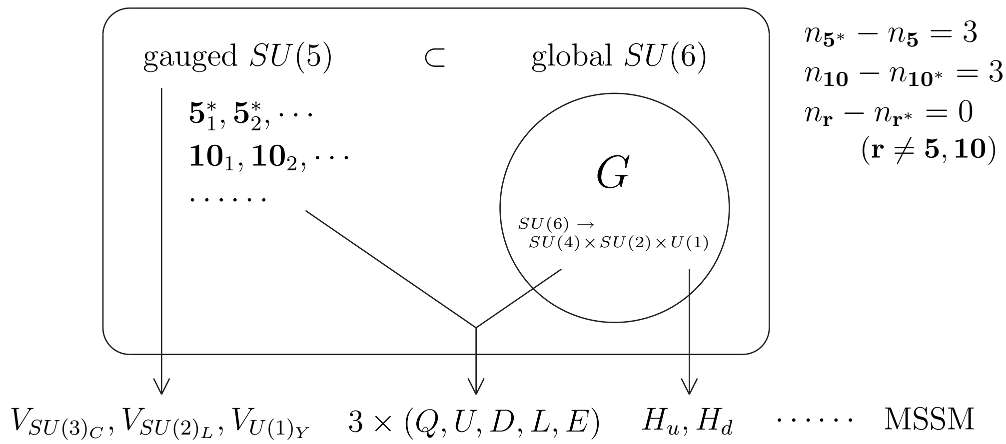

Figure 1: The basic picture of the theory in the 4D description.

naturally understand the origin of the hierarchical structure for the quark and lepton masses and mixings. The unwanted mass relations for the quarks and leptons can be avoided because low-energy quarks and leptons feel the GUT-breaking effects in the G sector.

Dangerous dimension four and five proton decay can be suppressed if the theory possesses a continuous or discrete R symmetry, under which the low-energy MSSM fields carry the charges $Q(1)$, $U(1)$, $D(1)$, $L(1)$, $E(1)$, $H_u(0)$ and $H_d(0)$ (and $N(1)$ if we introduce right-handed neutrino superfields N). This R symmetry is most likely spontaneously broken by the dynamics of the G sector (unless there is a low-energy singlet field that transforms nonlinearly under this symmetry; see discussion in section 4). The R symmetry should also be broken to the Z_2 subgroup, the R parity of the MSSM, in order to give weak scale masses to the gauginos. Supersymmetry breaking produces supersymmetric and supersymmetry-breaking masses for the Higgs doublets, as well as masses for the gauginos, squarks and sleptons, ensuring the stability of the desired vacuum. Successful supersymmetric gauge coupling unification is preserved if the threshold corrections associated with the G sector are sufficiently small.

We have depicted the basic picture of the theory in Fig. 1. How can we realize this picture in explicit models? It is not so straightforward to construct such models in the conventional 4D framework. In particular, it is not easy to find explicit gauge group and matter content for the G sector having all the features described above. (The difficulty increases if some of the relevant composite states are excited states of the G sector. We then cannot use beautiful exact results for $\mathcal{N} = 1$ supersymmetric gauge theories [19], which are applicable to lowest-lying modes.) In our framework, however, this problem is in some sense "bypassed." Suppose that the G sector possesses a large 't Hooft coupling, $\tilde{g}^2 \tilde{N}/16\pi^2 \gg 1$. In this case, the theory is so strongly coupled that the gauge theory description in terms of "gluons" and "quarks" does not make much sense. Instead, in this parameter region, the theory is better specified by composite "hadron" states, which have a tower structure. For $\tilde{N} \gg 1$, these "hadronic" tower states are weakly coupled [20], and under certain circumstances they can be identified as the Kaluza-Klein (KK) states of a weakly coupled higher dimensional theory. In particular, if the G sector is quasi-conformal ($\tilde{g}$ evolves very slowly) above its dynamical scale, the corresponding higher dimensional theory is formulated in warped AdS spacetime truncated by branes [13, 21]. In the next section we construct an explicit unified model in truncated 5D warped spacetime, which has all the features described in this section. In practice, once we have a theory in higher dimensions, we can forget about the "original" 4D picture for most purposes — our higher dimensional theory is an effective field theory with which we can consistently calculate various physical quantities. The theory does not require any more information than the gauge group, matter content, boundary conditions, and values of various parameters, to describe physics at energies below the cutoff scale M_* ($\gg M_U$).

3 Model

3.1 Basic symmetry structure

Following the general picture presented in the previous section, we consider 5D warped spacetime truncated by two branes: the UV and IR branes. The spacetime metric is given by

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \tag{1}$$

where y is the coordinate for the extra dimension and k denotes the inverse curvature radius of the warped AdS spacetime. The two branes are located at $y = 0$ (the UV brane) and $y = \pi R$ (the IR brane). This is the spacetime considered in Ref. [22], in which the AdS warp factor is used to generate the large hierarchy between the weak and the Planck scales by choosing the scales on the UV and IR branes to be the Planck and TeV scales, respectively ($kR \sim 10$). Here we choose instead the UV-brane and IR-brane scales to be $k \approx (10^{17} - 10^{18})$ GeV and $k' \equiv k\, e^{-\pi kR} \approx (10^{16} - 10^{17})$ GeV, respectively, so that the IR brane serves the role of breaking the unified symmetry. (A more detailed discussion on the determination of the scales is provided in later subsections.) In this sense, we may loosely call the UV and IR branes the Planck and GUT branes, respectively.

We consider supersymmetric unified gauge theory on this gravitational background. We choose the gauge symmetry in the bulk to be $SU(6)$, corresponding to the global symmetry that the dynamical GUT-breaking sector possesses in the 4D description of the model. The bulk $SU(6)$ gauge symmetry is broken to $SU(5) \times U(1)$ and $SU(4) \times SU(2) \times U(1)$ on the UV and

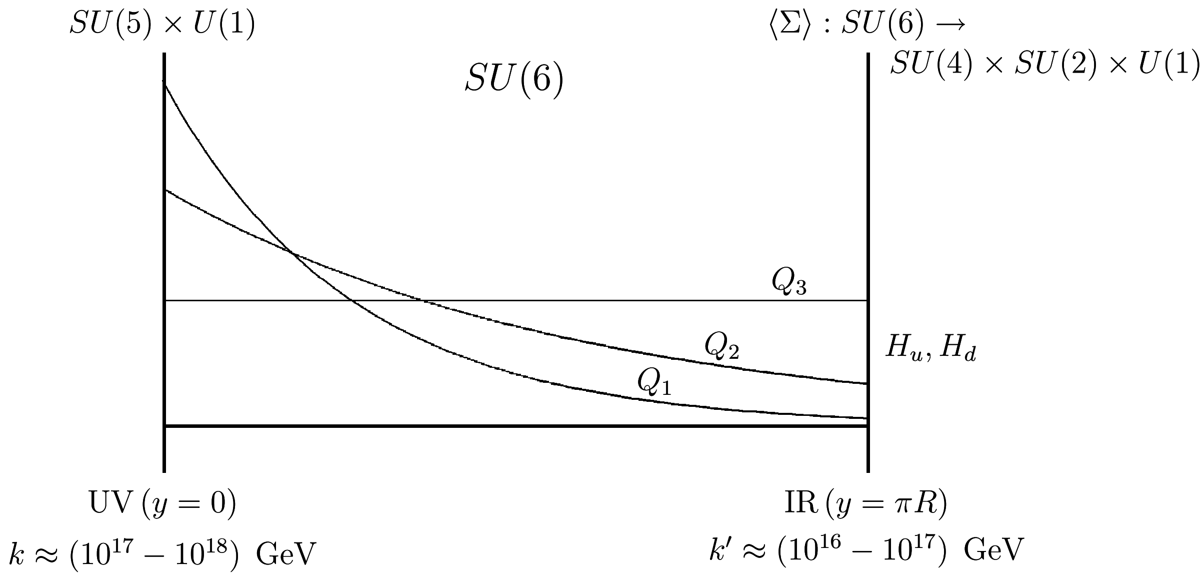

Figure 2: A schematic picture of the model in 5D.

IR branes, respectively, leaving an unbroken $SU(3) \times SU(2) \times U(1) \times U(1)$ gauge symmetry at low energies. There are two ways to break a gauge symmetry on a brane: by boundary conditions and by the Higgs mechanism. Let us first consider $SU(6) \to SU(5) \times U(1)$ on the UV brane. If this breaking is caused by the Higgs mechanism, then in the corresponding 4D description the fundamental gauge symmetry of the theory is $SU(6)$, which is spontaneously broken to $SU(5) \times U(1)$ at a very high energy $E \gg M_U$. In this case, we must introduce matter fields in representations of $SU(6)$, so that the standard $SU(5)$ embedding of matter fields [23] should be modified/extended. On the other hand, if $SU(6) \to SU(5) \times U(1)$ on the UV brane is caused by boundary conditions, then in the corresponding 4D description only the $SU(5) \times U(1)$ subgroup of the global $SU(6)$ symmetry is explicitly gauged (see Fig. 1), so that we can employ the standard $SU(5)$ embedding for matter fields. We thus adopt the latter option to construct our minimal model here, although models based on the former option can also be accommodated in our framework.

What about the symmetry breaking $SU(6) \to SU(4) \times SU(2) \times U(1)$ on the IR brane? If we break $SU(6)$ to $SU(4) \times SU(2) \times U(1)$ by boundary conditions on the IR brane, the two massless Higgs doublets, whose existence is guaranteed by the general symmetry argument presented in the previous section, arise from extra-dimensional components of the bulk $SU(6)$ gauge fields. This setup, however, leads to extra states lighter than $k' \approx (10^{16} - 10^{17})$ GeV once matter fields are introduced in the bulk with the zero modes localized towards the UV brane (such matter fields are used to naturally explain the observed hierarchies in the fermion masses and mixings; see subsection 3.3). These extra states generically do not fill complete $SU(5)$ representations and thus induce large threshold corrections for the standard model gauge couplings. Large threshold corrections can be avoided if we judiciously choose boundary conditions for matter fields, but bulk $SU(6)$ gauge invariance then still requires complicated structure for the matter sector to reproduce the observed fermion masses and mixings. These issues do not arise if the breaking $SU(6) \to SU(4) \times SU(2) \times U(1)$ is caused by the Higgs mechanism on the IR brane, as we will see later. We therefore adopt the Higgs breaking of $SU(6)$ on the IR brane. This completely determines the basic symmetry structure of our model, which is depicted in Fig. 2.

3.2 Gauge-Higgs sector and scales of the system

Let us start by describing the gauge-Higgs sector of the model. Using 4D $N = 1$ superfield language, in which the gauge degrees of freedom are contained in $V(A_\mu, \lambda)$ and $\Phi(\phi + iA_5, \lambda')$, the boundary conditions for the 5D $SU(6)$ gauge supermultiplet are given by

$$V: \left(\begin{array}{ccccc|c} (+,+) & (+,+) & (+,+) & (+,+) & (+,+) & (-,+) \\ (+,+) & (+,+) & (+,+) & (+,+) & (+,+) & (-,+) \\ (+,+) & (+,+) & (+,+) & (+,+) & (+,+) & (-,+) \\ (+,+) & (+,+) & (+,+) & (+,+) & (+,+) & (-,+) \\ (+,+) & (+,+) & (+,+) & (+,+) & (+,+) & (-,+) \\ \hline (-,+) & (-,+) & (-,+) & (-,+) & (-,+) & (+,+) \end{array} \right), \tag{2}$$

$$\Phi: \left(\begin{array}{ccccc|c} (-,-) & (-,-) & (-,-) & (-,-) & (-,-) & (+,-) \\ (-,-) & (-,-) & (-,-) & (-,-) & (-,-) & (+,-) \\ (-,-) & (-,-) & (-,-) & (-,-) & (-,-) & (+,-) \\ (-,-) & (-,-) & (-,-) & (-,-) & (-,-) & (+,-) \\ (-,-) & (-,-) & (-,-) & (-,-) & (-,-) & (+,-) \\ \hline (+,-) & (+,-) & (+,-) & (+,-) & (+,-) & (-,-) \end{array} \right), \tag{3}$$

where $+$ and $-$ represent Neumann and Dirichlet boundary conditions, respectively, and the first and second signs in parentheses represent boundary conditions at $y = 0$ and $y = \pi R$, respectively. These boundary conditions lead to $SU(6) \to SU(5) \times U(1)$ on the UV brane. Since only $(+,+)$ components have zero modes, we obtain 4D $N = 1$ $SU(5) \times U(1)$ gauge supermultiplets as massless fields at this point (coming from the upper left 5×5 block and the lower right element in V). All the other KK modes have masses of order $\pi k'$ or larger.

The symmetry breaking $SU(6) \to SU(4) \times SU(2) \times U(1)$ on the IR brane is caused by the vacuum expectation value (VEV) of a field $\Sigma(\mathbf{35})$ localized to the IR brane, where the number in the parenthesis represents the transformation property under $SU(6)$. We here consider that the Σ field is strictly localized on the IR brane and has the following superpotential:

$$\mathcal{L}_\Sigma = \delta(y - \pi R)\Big[\int d^2\theta \Big(\frac{M}{2}\,\mathrm{Tr}(\Sigma^2) + \frac{\lambda}{3}\,\mathrm{Tr}(\Sigma^3)\Big) + \mathrm{h.c.}\Big], \tag{4}$$

where the metric factor is absorbed into the normalization of the Σ field. (We will always absorb the metric factor into the normalizations of fields in similar expressions below, denoted by $\mathcal{L}$.) The field Σ is canonically normalized in 4D, so that natural values of the parameters M and λ are of order $M'_* = M_* e^{-\pi k R}$ and 4π, respectively. Here, M_* is the cutoff scale of the 5D theory. In general, the IR-brane potential for Σ also has higher dimension terms suppressed by M'_*, in addition to Eq. (4). The presence of these terms, however, does not affect the qualitative conclusions of our paper. Below, we assume that the parameter M is a factor of a few smaller than its naive size, e.g. $M \sim k'$, to make our analysis better controlled. In this case, the effect of higher dimension terms are expected to be suppressed even quantitatively.

The superpotential of Eq. (4) has the following vacuum:

$$\langle \Sigma \rangle \;=\; \mathrm{diag}\Big(-\frac{2M}{\lambda}, -\frac{2M}{\lambda}, \frac{M}{\lambda}, \frac{M}{\lambda}, \frac{M}{\lambda}, \frac{M}{\lambda}\Big), \tag{5}$$

where we have chosen $\lambda, M > 0$ without loss of generality. The VEV of Eq. (5) leads to $SU(6) \to SU(4) \times SU(2) \times U(1)$ on the IR brane, making a part of the $SU(5) \times U(1)$ gauge multiplet V massive. The remaining massless 4D $N = 1$ gauge multiplet is that of $SU(3) \times SU(2) \times U(1) \times U(1)$, which we identify as the standard model gauge group with an extra $U(1)_X$: $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$.

An important aspect of the model is that the vacuum of Eq. (5) is a part of a continuum of vacua, which can easily be seen by studying the excitations. Under the unbroken $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ gauge symmetry, the Σ field decomposes as

$$\begin{aligned}\Sigma \;=\; & \Sigma_G(\mathbf{8},\mathbf{1})_{(0,0)} + \Sigma_W(\mathbf{1},\mathbf{3})_{(0,0)} + \Sigma_B(\mathbf{1},\mathbf{1})_{(0,0)} \\ & + \Sigma_D(\mathbf{3}^*,\mathbf{1})_{(1/3,2)} + \Sigma_{\bar{D}}(\mathbf{3},\mathbf{1})_{(-1/3,-2)} + \Sigma_L(\mathbf{1},\mathbf{2})_{(-1/2,2)} + \Sigma_{\bar{L}}(\mathbf{1},\mathbf{2})_{(1/2,-2)} \\ & + \Sigma_X(\mathbf{3},\mathbf{2})_{(-5/6,0)} + \Sigma_{\bar{X}}(\mathbf{3}^*,\mathbf{2})_{(5/6,0)} + \Sigma_S(\mathbf{1},\mathbf{1})_{(0,0)},\end{aligned} \tag{6}$$

where the numbers in parentheses represent the quantum numbers under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$. The normalization of $U(1)_Y$ is chosen to match the conventional definition of hypercharge, while that of $U(1)_X$ is chosen, when matter fields are introduced, to match the conventional definition for the "$U(1)_\chi$" symmetry arising from $SO(10)/SU(5)$. Expanding the superpotential of Eq. (4) around the vacuum, we find that all the components of Σ obtain masses of order M except for Σ_X, $\Sigma_{\bar{X}}$, Σ_L and $\Sigma_{\bar{L}}$. Among these four, the first two are absorbed into the massive $SU(5)/321$ gauge fields, but the last two remain as massless chiral superfields, which parameterize the continuous degeneracy of vacua. This degeneracy is a consequence of the spontaneously broken $SU(6)$ symmetry, and the massless fields have the quantum numbers of a pair of Higgs doublets. We thus identify these fields as the two Higgs doublets of the MSSM: H_u and H_d.

We have found that the gauge-Higgs sector of our model gives only the 4D $N=1$ gauge supermultiplet for $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ and the two Higgs doublets H_u and H_d below the scale of order $M \sim k'$. The $U(1)_X$ gauge symmetry can be broken at a scale somewhat below k' by the Higgs mechanism. For example, we can introduce the superpotential on the UV brane

$$\mathcal{L}_X = \delta(y)\Bigl[\int d^2\theta\, Y(X\bar{X} - \Lambda^2) + \text{h.c.}\Bigr], \tag{7}$$

where Y, X and $\bar{X}$ are UV-brane localized chiral superfields that are singlet under $SU(5)$ and have charges of 0, 10 and -10 under $U(1)_X$, respectively. The scale Λ is that for $U(1)_X$ breaking, which may be generated by some other dynamics. The superpotential of Eq. (7) produces the VEVs $\langle X\rangle = \langle\bar{X}\rangle = \Lambda$, leading to $U(1)_X$ breaking at the scale Λ. The nonvanishing VEV for $\bar{X}$ can also be used to generate small neutrino masses through the conventional seesaw mechanism, as we will see later. This motivates the values of the X and $\bar{X}$ charges.

Various scales of our system – the AdS inverse curvature radius k, the size of the extra dimension R, and the cutoff scale of the effective 5D theory M_* – are constrained by the scale of gauge coupling unification, the size of the unified gauge coupling $g_U \simeq 0.7$, and the value of the 4D (reduced) Planck scale $M_{\rm Pl} \simeq 2.4 \times 10^{18}$ GeV. In our warped 5D theory, it is natural to consider that parameters in the bulk and on the IR brane obey naive dimensional analysis (at least roughly) while those on the UV brane do not, because the former represent strongly coupled G dynamics while the latter represent the weakly coupled elementary sector. Using naive dimensional analysis in higher dimensions [24], we obtain the following Lagrangian for the graviton and the gauge fields:

$$\mathcal{L} \approx \delta(y)\left[\frac{\tilde{M}^2}{2}\mathcal{R}^{(4)} - \frac{1}{4\tilde{g}^2}F^{\mu\nu}F_{\mu\nu}\right] + \left[\frac{1}{2}\frac{M_*^3}{16\pi^3}\mathcal{R}^{(5)} - \frac{1}{4}\frac{CM_*}{16\pi^3}F^{MN}F_{MN}\right] \tag{8}$$

where $\mathcal{R}^{(4)}$ and $\mathcal{R}^{(5)}$ are the 4D and 5D Ricci curvatures, respectively, $M,N = 0,1,2,3,5$, and C is a group theoretical factor, $C \simeq 6$. This leads to the following relations:

$$\frac{1}{g_U^2} \simeq \frac{1}{\tilde{g}^2} + \frac{C}{16\pi^2}\left(\frac{M_*}{\pi k}\right)\pi k R, \tag{9}$$

$$M_{\rm Pl}^2 \simeq \tilde{M}^2 + \frac{k^2}{16}\left(\frac{M_*}{\pi k}\right)^3. \tag{10}$$

Now, gauge coupling unification at $M_U \approx 10^{16}$ GeV implies that we should choose M to be around this scale, and thus $k' = k\,e^{-\pi k R} \approx (10^{16} - 10^{17})$ GeV. Then, choosing $M_*/\pi k$ to be a factor of a few, e.g. $M_*/\pi k \simeq (2 \sim 3)$, to make the higher dimensional description trustable, we obtain $k \lesssim 10^{18}$ GeV from Eq. (10) (and $\tilde{M}^2 > 0$). We thus find that the scales of our 5D theory should be chosen as $k \approx (10^{17} - 10^{18})$ GeV and $k' \approx (10^{16} - 10^{17})$ GeV, which implies $kR \sim 1$, with the cutoff scale M_* a factor of a few larger than πk. The UV-brane gauge coupling $\tilde{g}$ is then likely to be nonzero, implying that the elementary $SU(5)$ gauge field has nonvanishing tree-level kinetic terms in the 4D description. In particular, this implies that elementary $SU(5)$ gauge interactions are likely to be weakly coupled at energies $E \gg M_U$.

3.3 Matter sector and quark and lepton masses and mixings

Let us now include matter fields in the model. In the 4D description of the theory, low-energy quark and lepton fields arise from mixtures of elementary states, which transform as **10**'s and $\mathbf{5}^*$'s under the gauged $SU(5)$, and composite states of G, which form multiplets of the global $SU(6)$. In the 5D theory, this situation is realized by introducing matter hypermultiplets in the bulk, which are representations of $SU(6)$, and by imposing $SU(6)$-violating boundary conditions on the UV brane. We here present an explicit realization of this picture, leading to realistic phenomenology at low energies.

We begin by considering the structure of the matter sector for a single generation. For quarks and leptons that are incorporated into the **10** representation of $SU(5)$, $\{Q, U, E\}$, we introduce a bulk hypermultiplet $\{\mathcal{T}, \mathcal{T}^c\}$ transforming as **20** under $SU(6)$:

$$\mathcal{T}(\mathbf{20}) = \mathbf{10}_1^{(+,+)} \oplus \mathbf{10}_{-1}^{*(-,+)}, \tag{11}$$

$$\mathcal{T}^c(\mathbf{20}) = \mathbf{10}_{-1}^{*(-,-)} \oplus \mathbf{10}_1^{(+,-)}, \tag{12}$$

where $\mathcal{T}$ and $\mathcal{T}^c$ represent 4D $N=1$ chiral superfields that form a hypermultiplet in 5D. (Our notation is such that "non-conjugated" and "conjugated" chiral superfields have the opposite gauge quantum numbers; see e.g. [25]. They have the same quantum numbers for **20** of $SU(6)$ because **20** is a (pseudo-)real representation.) The right-hand-side of Eqs. (11, 12) shows the decomposition of $\mathcal{T}$ and $\mathcal{T}^c$ into representations of $SU(5) \times U(1)_X$ (in an obvious notation), as well as the boundary conditions imposed on each component (in the same notation as that in Eqs. (2, 3)). With these boundary conditions, the only massless state arising from $\{\mathcal{T}, \mathcal{T}^c\}$ is $\mathbf{10}_1$ of $SU(5) \times U(1)_X$ from $\mathcal{T}$, which we identify as the low-energy quarks and leptons Q, U and E.

A bulk hypermultiplet $\{\mathcal{H}, \mathcal{H}^c\}$ can generically have a mass term in the bulk, which is written as

$$S = \int d^4x \int_0^{\pi R} dy \left[e^{-3k|y|} \int d^2\theta \, c_{\mathcal{H}} \, k \, \mathcal{H}\mathcal{H}^c + \text{h.c.} \right], \tag{13}$$

in the basis where the kinetic term is given by $S_{\rm kin} = \int d^4x \int dy \, [e^{-2k|y|} \int d^4\theta \, (\mathcal{H}^\dagger \mathcal{H} + \mathcal{H}^c \mathcal{H}^{c\dagger}) + \{e^{-3k|y|} \int d^2\theta \, (\mathcal{H}^c \partial_y \mathcal{H} - \mathcal{H} \partial_y \mathcal{H}^c)/2 + \text{h.c.}\}]$ [26]. The parameter $c_{\mathcal{H}}$ controls the wavefunction profile of the zero mode. For $c_{\mathcal{H}} > 1/2$ ($< 1/2$) the wavefunction of a zero mode arising from $\mathcal{H}$ is localized to the UV (IR) brane; for $c_{\mathcal{H}} = 1/2$ it is conformally flat. (If a zero mode arises from $\mathcal{H}^c$, its wavefunction is localized to the IR (UV) brane for $c_{\mathcal{H}} > -1/2$ ($< -1/2$) and conformally flat for $c_{\mathcal{H}} = -1/2$.) We choose these c parameters to take values larger than about 1/2 for matter fields. For these values of c parameters, all the KK excited states of $\{\mathcal{T}, \mathcal{T}^c\}$ have masses of order $\pi k'$ or larger, so that the $\{\mathcal{T}, \mathcal{T}^c\}$ multiplet gives only the massless $\mathbf{10}_1$ state below the energy scale of k'.

For quarks and leptons incorporated into the $\mathbf{5}^*$ representation of $SU(5)$, $\{D, L\}$, we introduce a bulk hypermultiplet $\{\mathcal{F}, \mathcal{F}^c\}$ transforming as $\mathbf{70}^*$ under $SU(6)$:

$$\mathcal{F}(\mathbf{70}^*) \;=\; \mathbf{5}^{*(+,+)}_{-3} \oplus \mathbf{10}^{*(-,+)}_{-1} \oplus \mathbf{15}^{*(-,+)}_{-1} \oplus \mathbf{40}^{*(-,+)}_{1}, \tag{14}$$

$$\mathcal{F}^c(\mathbf{70}) \;=\; \mathbf{5}^{(-,-)}_{3} \oplus \mathbf{10}^{(+,-)}_{1} \oplus \mathbf{15}^{(+,-)}_{1} \oplus \mathbf{40}^{(+,-)}_{-1}, \tag{15}$$

where the right-hand-side again shows the decomposition into representations of $SU(5) \times U(1)_X$, together with the boundary conditions imposed on each component.[1] With these boundary conditions, the only massless state arising from $\{\mathcal{F}, \mathcal{F}^c\}$ is $\mathbf{5}^*_{-3}$ of $SU(5) \times U(1)_X$ from $\mathcal{F}$, which we identify as the low-energy quarks and leptons D and L. All the KK excited states have masses of order $\pi k'$ or larger for $c_{\mathcal{F}} \gtrsim 1/2$.

The right-handed neutrino N arises from a bulk hypermultiplet $\{\mathcal{N}, \mathcal{N}^c\}$ transforming as $\mathbf{56}$ of $SU(6)$:

$$\mathcal{N}(\mathbf{56}) \;=\; \mathbf{1}^{(+,+)}_{5} \oplus \mathbf{5}^{(-,+)}_{3} \oplus \mathbf{15}^{(-,+)}_{1} \oplus \mathbf{35}^{(-,+)}_{-1}, \tag{16}$$

$$\mathcal{N}^c(\mathbf{56}^*) \;=\; \mathbf{1}^{(-,-)}_{-5} \oplus \mathbf{5}^{*(+,-)}_{-3} \oplus \mathbf{15}^{*(+,-)}_{-1} \oplus \mathbf{35}^{*(+,-)}_{1}. \tag{17}$$

The zero mode arises only from $\mathbf{1}_5$ in $\mathcal{N}$, which is identified as the right-handed neutrino supermultiplet N. The other KK states are all heavier than of order $\pi k'$ for $c_{\mathcal{N}} \gtrsim 1/2$.

The Yukawa couplings for the quarks and leptons arise from IR-brane localized terms

$$\mathcal{L}_{\rm Yukawa} = \delta(y - \pi R) \Big[\int d^2\theta \Big(y_{\mathcal{T}} \mathcal{T}\mathcal{T}\Sigma + y_{\mathcal{F}} \mathcal{T}\mathcal{F}\Sigma + y_{\mathcal{N}} \mathcal{F}\mathcal{N}\Sigma \Big) + {\rm h.c.} \Big]. \tag{18}$$

[1] Note that the signs $\pm$ for the boundary conditions in Eqs. (14, 15) represent the Neumann/Dirichlet boundary conditions in the interval $y : [0, \pi R]$. In the orbifold picture, the boundary conditions of Eqs. (14, 15) can be obtained effectively as follows. We prepare a hypermultiplet obeying the boundary conditions $\mathcal{F}(\mathbf{70}^*) = \mathbf{5}^{*(+,+)}_{-3} \oplus \mathbf{10}^{*(-,+)}_{-1} \oplus \mathbf{15}^{*(-,+)}_{-1} \oplus \mathbf{40}^{*(+,+)}_{1}$ and $\mathcal{F}^c(\mathbf{70}) = \mathbf{5}^{(-,-)}_{3} \oplus \mathbf{10}^{(+,-)}_{1} \oplus \mathbf{15}^{(+,-)}_{1} \oplus \mathbf{40}^{(-,-)}_{-1}$, where the first and second signs in the parentheses represent transformation properties under the reflection $y \leftrightarrow -y$ and $(y - \pi R) \leftrightarrow -(y - \pi R)$, respectively. We then introduce a UV-brane localized chiral superfield transforming as $\mathbf{40}_{-1}$ under $SU(5) \times U(1)_X$, and couple it to the $\mathbf{40}^{*(+,+)}_{1}$ state from $\mathcal{F}(\mathbf{70}^*)$. This reproduces the boundary conditions of Eqs. (14, 15) in the limit that this coupling (brane mass term) becomes large. (For the relation between a large brane mass term and the Dirichlet boundary condition, see e.g. [27].) The fact that the boundary conditions of Eqs. (14, 15) can be reproduced in the orbifold picture by taking a consistent limit guarantees their consistency. In the 4D description, this corresponds to introducing only a $\mathbf{5}^*_{-3}$ elementary state, which couples to a component of a G-invariant operator transforming as $\mathbf{70}$ under the global $SU(6)$. Similar remarks also apply to other fields, e.g. the $\{\mathcal{N}, \mathcal{N}^c\}$ hypermultiplet in Eqs. (16, 17).

(The Yukawa couplings also receive contributions from higher dimension terms as will be seen later in this subsection.) Note that these interactions, as well as those in Eq. (4), respect the usual R parity of the MSSM, with Σ even.

The interactions of Eq. (18) give the Yukawa couplings of the quark and lepton chiral superfields, Q, U, D, L, E and N, with the Higgs doublets, H_u and H_d, at low energies ($W = QUH_u$, $QDH_d + LEH_d$ and LNH_u from the first, second and third terms, respectively). Recall that the two Higgs doublets of the MSSM, H_u and H_d, arise from Σ as pseudo-Goldstone chiral superfields of the broken $SU(6)$ symmetry. For matter fields with $|c| > 1/2$, the Yukawa couplings receive suppressions due to the fact that the fields effectively feel only the IR brane (strong dynamics) or the UV brane (explicit breaking of $SU(6)$), both of which are needed to generate nonvanishing Yukawa couplings at low energies [18]. Then, considering that $y_{\mathcal{T}} \sim y_{\mathcal{F}} \sim y_{\mathcal{N}} = O(4\pi^2/M'_*)$ from naive dimensional analysis, we find that the low-energy Yukawa coupling y arising from the IR-brane term $\int d^2\theta\, \mathcal{M}_1 \mathcal{M}_2 \Sigma$ ($\mathcal{M}_1, \mathcal{M}_2 = \mathcal{T}, \mathcal{F}, \mathcal{N}$) takes a value

$$y \approx 4\pi f_1 f_2 \left(\frac{\pi k}{M_*} \right), \tag{19}$$

where $f_i \simeq (k'/k)^{|c_{\mathcal{M}_i}|-1/2}$ for $|c_{\mathcal{M}_i}| > 1/2$ and $f_i \simeq 1$ for $|c_{\mathcal{M}_i}| < 1/2$ ($i = 1, 2$); for $|c_{\mathcal{M}_i}| \simeq 1/2$, f_i receives a logarithmic suppression, $f_i \simeq 1/(\ln(k/k'))^{1/2}$. This allows us to explain the observed hierarchies of fermion masses and mixings by powers of $k'/k = e^{-\pi kR} = O(0.1)$, by choosing different values of $c_{\mathcal{T}}, c_{\mathcal{F}}$ and $c_{\mathcal{N}}$ for different generations. This is similar to the situation where the hierarchies are explained by overlaps of matter and Higgs wavefunctions [28, 12], although in the present setup the low-energy Yukawa couplings are also suppressed for $c_{\mathcal{M}_i} < -1/2$, where apparent overlaps between matter and Higgs fields are large, due to the pseudo-Goldstone boson nature of the Higgs doublets. This opens the possibility of localizing the first two generations to the IR brane, rather than to the UV brane as we will do shortly, to generate the observed hierarchies of fermion masses and mixings.

The right-handed neutrino superfield N can obtain a large mass term through the UV-brane operator

$$\mathcal{L}_N = \delta(y) \left[\int d^2\theta\, \frac{\eta}{2} \bar{X} N^2 + \text{h.c.} \right], \tag{20}$$

where $\bar{X}$ is a $U(1)_X$-breaking field, having the VEV $\langle \bar{X} \rangle = \Lambda$ (see Eq. (7)). This gives a small mass for the observed left-handed neutrino through the conventional seesaw mechanism [29].[2]

It is rather straightforward to generalize the analysis so far to the case of three generations. We simply introduce a set of bulk hypermultiplets $\{\mathcal{T}, \mathcal{T}^c\}$, $\{\mathcal{F}, \mathcal{F}^c\}$ and $\{\mathcal{N}, \mathcal{N}^c\}$ for each generation. The couplings $y_{\mathcal{T}}$, $y_{\mathcal{F}}$ and $y_{\mathcal{N}}$ in Eq. (18) and η in Eq. (20) then become 3×3

[2] An alternative possibility to generate a small neutrino mass is to strongly localize the N field to the IR brane by taking $c_{\mathcal{N}} \ll -1/2$, in which case the neutrino Yukawa coupling is strongly suppressed and we can obtain a small Dirac neutrino mass. The scale of the neutrino mass, however, is unexplained in this case.

matrices. We assume that there is no special structure in these matrices, so that all the elements in $y_{\mathcal{T}}$, $y_{\mathcal{F}}$ and $y_{\mathcal{N}}$ are of order $4\pi^2/M'_*$, suggested by naive dimensional analysis. The observed fermion masses and mixings, however, can still be reproduced through the dependence of the low-energy Yukawa couplings on the values of bulk hypermultiplet masses $c_{\mathcal{T}}, c_{\mathcal{F}}$ and $c_{\mathcal{N}}$. Let us take, for example, the bulk masses to be

$$c_{\mathcal{T}_1} \simeq \frac{5}{2}, \quad c_{\mathcal{T}_2} \simeq \frac{3}{2}, \quad c_{\mathcal{T}_3} \simeq \frac{1}{2}, \quad c_{\mathcal{F}_1} \simeq c_{\mathcal{F}_2} \simeq c_{\mathcal{F}_3} \simeq \frac{3}{2}, \quad c_{\mathcal{N}_1} \simeq c_{\mathcal{N}_2} \simeq c_{\mathcal{N}_3} \simeq \frac{1}{2}. \tag{21}$$

Then, taking $M_*/\pi k$ to be a factor of a few, e.g. $2 \sim 3$, we obtain the following low-energy Yukawa matrices from Eq. (19):

$$y_u \approx \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}, \quad y_d \approx y_e^T \approx \epsilon \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon & \epsilon & \epsilon \\ 1 & 1 & 1 \end{pmatrix}, \quad y_\nu \approx \epsilon \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{22}$$

where y_u, y_d, y_e and y_ν are defined in the low-energy superpotential by

$$W = (y_u)_{ij} Q_i U_j H_u + (y_d)_{ij} Q_i D_j H_d + (y_e)_{ij} L_i E_j H_d + (y_\nu)_{ij} L_i N_j H_u, \tag{23}$$

with $i, j, = 1, 2, 3$, and

$$\epsilon \equiv \frac{k'}{k} \simeq \frac{1}{20} \quad \text{for } \; kR \simeq 1. \tag{24}$$

Together with a structureless Majorana mass matrix for the right-handed neutrinos, $M_N = \eta \langle \bar{X} \rangle$, the Yukawa matrices of Eq. (22) well reproduces gross features of the observed quark and lepton masses and mixings [30]. It is straightforward to make further refinements on this basic picture; for example, we can make $c_{\mathcal{F}_1}$ somewhat larger than $3/2$ to better reproduce down-type quark and charged lepton masses, as well as the neutrino mixing angles θ_{12} and θ_{13}. A schematic picture for the zero-mode wavefunctions (for the $\{\mathcal{T}, \mathcal{T}^c\}$ multiplets) is depicted in Fig. 2.

Unwanted $SU(5)$ mass relations for the first two generation fermions can be avoided by using higher dimension operators, e.g. of the form $\mathcal{L} \sim \delta(y - \pi R) \int d^2\theta\, \mathcal{T}\mathcal{F}\Sigma^2$. (Violation of $SU(5)$ relations may also come from $SU(5)$-violating mixings between the matter zero modes and the corresponding KK excited states, arising from the IR-brane terms of Eq. (18) through the Σ VEV.) Since the effects are higher order in $\langle \Sigma \rangle / M'_*$, which we assume somewhat small, $O(1)$ violation in the Yukawa coupling requires a somewhat suppressed coefficient for the leading $SU(5)$-invariant piece coming from Eq. (18). A realistic pattern for the fermion masses and mixings can be obtained if (only) the 22 element of the $y_{\mathcal{F}}$ matrix is somewhat suppressed [31].

The three generation model allows IR-brane operators of the form $\mathcal{L} \sim \delta(y - \pi R) \int d^2\theta\, \epsilon^{ij} \mathcal{T}_i \mathcal{T}_j$, where the antisymmetry in the generation indices i, j arises from the pseudo-real nature of the **20** representation. The existence of these operators, however, does not significantly affect predictions of the model.

To summarize, we have obtained an $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ gauge theory below the scale of $M \sim k'$, with three generations of matter, Q, U, D, L, E and N, and two Higgs doublets, H_u and H_d. The Yukawa couplings of Eq. (23) are obtained with realistic patterns for quark and lepton masses and mixings. The $U(1)_X$ gauge symmetry is spontaneously broken at the scale Λ, somewhat below k', giving masses to the right-handed neutrino superfields of order Λ. We thus have the complete MSSM, supplemented by seesaw neutrino masses, below the unification scale $\sim k'$. We emphasize that the successes of our model depend only on its basic features, such as the symmetry structure and locations of fields. They are thus quite robust. For example, the existence of higher dimension operators in the IR-brane potential, e.g. terms of the form $\mathrm{Tr}(\Sigma^n)$ (n: integers > 3) added to Eq. (4), does not destroy these successes.

3.4 Gauge coupling unification and proton decay

In this subsection, we present a study on proton decay and gauge coupling unification in our model, to demonstrate that it can accommodate realistic phenomenology at low energies. In this subsection we consider matter configurations such that lighter generations are localized more towards the UV brane, as in the example of Eq. (21).

We first note that the terms in Eqs. (18) introduce, through the VEV of Σ, $SU(5)$-violating mass splittings into the KK towers for the matter fields: $\{\mathcal{T}, \mathcal{T}^c\}$, $\{\mathcal{F}, \mathcal{F}^c\}$ and $\{\mathcal{N}, \mathcal{N}^c\}$. These splittings, in turn, give threshold corrections to gauge coupling unification. Similar corrections also arise from the gauge KK towers. We expect, however, that these corrections are not large. Using the AdS/CFT correspondence, we estimate the size of the corrections to be of order $(C/16\pi^2)(M_*/\pi k)$ for $1/g_a^2$, where g_a are the 4D gauge couplings. Moreover, if the value of $\langle\Sigma\rangle$ (and thus M) is somewhat suppressed compared with its naive size of $M'_*/4\pi$, as we assume here, the threshold corrections receive additional suppressions of $O(4\pi\langle\Sigma\rangle/M'_*)$ because the spectrum of the KK towers becomes $SU(5)$ symmetric for $4\pi\langle\Sigma\rangle/M'_* \ll 1$. The contributions from tree-level IR-brane operators, such as $\int d^2\theta\, \Sigma \mathcal{W}^\alpha \mathcal{W}_\alpha$, are also sufficiently small, of order $C/16\pi^2$ for $1/g_a^2$ with an additional suppression of $O(4\pi\langle\Sigma\rangle/M'_*)$ for small $\langle\Sigma\rangle$.

Another important issue in supersymmetric unified theories is dimension five proton decay caused by low-energy operators of the form $W \sim QQQL$, $UUDE$. There are two independent sources for these operators: tree-level operators existing at the gravitational scale and operators generated by the GUT (breaking) dynamics. In our theory, the former correspond to tree-level operators $\mathbf{10}_1\mathbf{10}_1\mathbf{10}_1\mathbf{5}^*_{-3} \supset QQQL$, $UUDE$ located on the UV brane, where the subscripts on $\mathbf{10}_1 \subset \mathcal{T}$ and $\mathbf{5}^*_{-3} \subset \mathcal{F}$ denote the $U(1)_X$ charges. While the coefficients of these operators are suppressed by the fundamental scale M_*, which is larger than the unification scale, it is still problematic, especially because we do not have any Yukawa suppressions in the coefficients. Therefore, to suppress these contributions, we impose a discrete $Z_{4,R}$ symmetry on the theory,

	V	Φ	Σ	$\mathcal{T}$	$\mathcal{T}^c$	$\mathcal{F}$	$\mathcal{F}^c$	$\mathcal{N}$	$\mathcal{N}^c$	Y	X	$\bar{X}$
$Z_{4,R}$	0	0	0	1	1	1	1	1	1	2	0	0

Table 1: $Z_{4,R}$ charges for fields.

whose charge assignment is given in Table 1 (in the normalization that the R charge of the superpotential is 2). As is clear from the terms in Eq. (4), this symmetry should be broken on the IR brane, i.e. broken by the dynamics of the GUT breaking sector G. This can be incorporated by introducing a spurion chiral superfield ϕ with $\langle\phi\rangle \sim M'_*/4\pi$ on the IR brane, whose $Z_{4,R}$ charge is $+2$, or equivalently introducing fields ϕ and $\bar{\phi}$ of $Z_{4,R}$ charges $+2$ and -2 with the superpotential giving the VEVs for these fields. This introduces "$O(1)$" breaking of $Z_{4,R}$ on the IR brane, keeping $Z_{4,R}$ invariance for the UV-brane terms.[3]

After killing the UV-brane operators, the low-energy dimension five proton decay operators can still be generated through strong G dynamics, since the $Z_{4,R}$ symmetry is spontaneously broken by this dynamics. One source is tree-level dimension five operators on the IR brane. These operators, however, receive suppressions of order the Yukawa couplings in 4D, because the wavefunctions for light generation matter are suppressed on the IR brane due to the bulk hypermultiplet masses, and so are not particularly dangerous. (In the 4D picture, these suppressions arise from small mixings between the elementary and composite matter states for light generations.) The only potentially dangerous contribution to dimension five proton decay in our model then comes from the exchange of the colored triplet Higgsinos – composite states of G arising as components of Σ – because the mass of these states can be smaller than M'_*. To suppress this contribution, we can simply raise the mass of the colored triplet Higgsino states compared with the unification scale; in fact, the mass is expected to be larger than the GUT breaking VEV because the coupling λ in Eq. (4) is naturally of order 4π. Note that because of the existence of threshold corrections from KK towers to gauge coupling unification, there is no tight relation between the mass of the triplet Higgsinos and the low-energy values of the gauge couplings, which excluded the minimal SUSY $SU(5)$ GUT in 4D [2].

[3]The $Z_{4,R}$ symmetry can be gauged in 5D if we cancel the discrete $Z_{4,R}$-$SU(5)^2$ anomaly by the Green-Schwarz mechanism [32], by introducing a singlet field S that transforms nonlinearly under $Z_{4,R}$ and couples to the $SU(5)$ gauge kinetic term on the UV brane. We consider that the S field appears only in front of the kinetic term of the $SU(5)$ gauge superfields, and not in UV-brane superpotential terms. Such terms would potentially induce dimension five proton decay, although they are suppressed in a certain (broad) region for the S VEV.

3.5 Supersymmetry breaking

Our model can be combined with almost any supersymmetry breaking scenario. If the mediation scale of supersymmetry breaking is lower than the unification scale, there are essentially no particular implications from our theory on the pattern of supersymmetry breaking. On the other hand, if the mediation scale is higher, there can be interesting implications, e.g., on the flavor structure of supersymmetry breaking masses. For example, if the supersymmetry breaking sector is localized on the IR brane, i.e. arises as a result of the dynamics of G, the third generation superparticles (presumably only the ones coming from the **10** representation of $SU(5)$) can have different masses than the lighter generation superparticles, which receive universal masses from the gauginos through loop corrections [33].[4] These are consequences of our way of generating hierarchies in fermion masses and mixings.

The supersymmetric mass (the μ term) and supersymmetry-breaking masses (the μB term and non-holomorphic scalar squared masses) for the Higgs doublets are both generated through supersymmetry breaking. In the case that the supersymmetry breaking sector is localized on or directly communicates with the IR brane, these masses are generated through IR-brane operators of the form, $\mathcal{L} \sim \delta(y-\pi R) \int d^2\theta \, \{ZM\Sigma^2/M'_* + Z\Sigma^3/M'_*\} + \text{h.c.}$ and $\delta(y-\pi R) \int d^4\theta \, \{(Z + Z^\dagger)\Sigma^\dagger\Sigma/M'_* + Z^\dagger Z(\Sigma^2 + \Sigma^{\dagger 2})/M'^2_* + Z^\dagger Z \Sigma^\dagger \Sigma/M'^2_*\}$, where Z is a chiral superfield responsible for supersymmetry breaking, $\langle Z \rangle = \theta^2 F_Z$, and we have omitted $O(1)$ coefficients. These operators produce supersymmetry breaking terms in the Σ potential, which lead to a slight shift of the vacuum from Eq. (5) and consequently generate weak scale masses for components of H_u and H_d. The generated masses respect the relation

$$\mu B = \left| |\mu|^2 + m_{H_u}^2 \right|, \qquad m_{H_u}^2 = m_{H_d}^2, \tag{25}$$

reflecting the fact that the scalar potential for Σ still has a global $SU(6)$ symmetry, where $m_{H_u}^2$ and $m_{H_d}^2$ are non-holomorphic supersymmetry breaking squared masses for H_u and H_d, and we have taken the phase convention that $\mu B > 0$. Note that, unlike the case where the Higgs fields are non pseudo-Goldstone fields [34], the Kähler potential terms of the form $\delta(y-\pi R) \int d^4\theta \, \Sigma^2$, $\delta(y-\pi R) \int d^4\theta \, \{Z^\dagger \Sigma^2/M'_* + \text{h.c.}\}$ and $\delta(y-\pi R) \int d^4\theta \, \{Z^\dagger Z \Sigma^2/M'^2_* + \text{h.c.}\}$ do not produce a weak scale μ term; we need supersymmetry breaking interactions for Σ, generated by superpotential terms or $\delta(y-\pi R) \int d^4\theta \, (Z + Z^\dagger)\Sigma^\dagger\Sigma/M'_*$.

An interesting case arises if supersymmetry is broken in a hidden sector that does not have direct interactions with the GUT breaking sector. In this case, the Higgs sector supersymmetry breaking parameters arise through gravitational effects and obey the tighter relation

$$\mu B = |\mu|^2, \qquad m_{H_u}^2 = m_{H_d}^2 = 0. \tag{26}$$

[4] We thank R. Kitano for discussions on this issue.

In the language of the compensator formalism (see e.g. [35]), these terms arise from $\mathcal{L} \sim \delta(y - \pi R) \int d^2\theta\, \phi (M/2)\Sigma^2 + \text{h.c.}$, where $\phi = 1 + \theta^2 m_{3/2}$ is the compensator field with $m_{3/2}$ the gravitino mass. (Here, we have assumed that supersymmetry breaking in the compensator field is not canceled by the conformal dynamics of the GUT breaking sector.) The relations of Eqs. (25, 26) hold at the unification scale of $O(k')$, so that their connections to low energy parameters must involve renormalization group effects between the unification and the weak scales. It is also possible that there are additional contributions to supersymmetry breaking parameters, e.g. $m_{H_u}^2$ and $m_{H_d}^2$, in addition to the ones in Eqs. (25, 26)

Alternatively, the μ and μB terms may be generated below the unification scale. For example, they may be generated associated with the dynamics of $U(1)_X$ breaking [36]. In this case there is no trace in the Higgs sector parameters that the Higgs fields are pseudo-Goldstone fields of the GUT breaking dynamics.

4 Other Theories: GUT Engineering on the IR Brane

So far, we have considered a theory in which the lightness of the two Higgs doublets is understood by the pseudo-Goldstone mechanism associated with the dynamics of GUT breaking. As we have seen, this can be elegantly implemented in our framework by considering the bulk $SU(6)$ gauge symmetry, broken to $SU(5) \times U(1)$ and $SU(4) \times SU(2) \times U(1)$ on the UV and IR branes, respectively. The mass of the light Higgs doublets is protected from the existence of explicit breaking by localizing these fields ($\subset \Sigma$) on the IR brane, which is geometrically separated from the UV brane where explicit breaking of $SU(6)$ resides. In the 4D description of the model, the global $SU(6)$ symmetry of the GUT breaking sector is understood as a "flavor" symmetry of this sector, and the extreme suppression of explicit symmetry breaking effects in the Σ potential comes from the fact that Σ is a composite field, with the corresponding operator having a (very) large canonical mass dimension. An interesting thing about our construction is that it allows us to implement these mechanisms in simple and controllable ways in effective field theory, giving a simple and calculable unified theory above M_U in which the lightness of the Higgs doublets is understood by a symmetry principle.

Let us now consider if we can construct simpler theories in our warped space framework. Suppose we consider a supersymmetric $SU(5)$ gauge theory in the 5D warped spacetime of Eq. (1), and suppose that the bulk $SU(5)$ gauge symmetry is broken to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ subgroup on the IR brane by boundary conditions. In this case, the doublet Higgs fields may be light without being accompanied by their triplet partners if they propagate in the bulk with appropriate boundary conditions imposed at the GUT breaking brane [8, 9], or if they are simply located on that brane [37]. In the 4D description, however, this seems to be simply "postulating" particular dynamics of the GUT breaking sector that splits the mass of the doublet components from that of the triplet partners, and it is not clear if this can be regarded as a "solution" to the doublet-triplet splitting problem. For example, we have a continuous parameter, the tree-level mass of the Higgs doublets on the IR brane, that has to be chosen to be very small to achieve the splitting. The situation may be better if this parameter is forbidden by a symmetry, e.g. an R symmetry [9]. This symmetry may be imposed as a global symmetry in 5D, but in that case it is not entirely clear if such a symmetry is preserved by strong G dynamics (5D quantum gravity effects). To avoid this and to give a non-trivial meaning to the symmetry in the context of gauge/gravity duality, we can gauge the symmetry in higher dimensions (although it can still be broken on the UV brane, eliminating the existence of the corresponding gauge field in 4D). In this case, anomaly cancellation conditions become an issue, and we find that for a continuous $U(1)_R$ or a discrete $Z_{4,R}$ symmetry (with the charge assignment given by $V_{SU(5)}(0)$, $T_{\mathbf{10}}(1)$, $F_{\mathbf{5}^*}(1)$, $N_{\mathbf{1}}(1)$, $H_{\mathbf{5}}(0)$, $\bar{H}_{\mathbf{5}^*}(0)$, assuming the MSSM matter content at low energies) we need to cancel the low energy anomalies via the Green-Schwarz mechanism [32]. This requires the introduction of a singlet field S on the IR brane which transforms nonlinearly under the R symmetry and couples to the $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ gauge kinetic terms with appropriate coefficients. (Anomaly transmission across the bulk [38] may also be necessary to make the full 5D theory anomaly free, depending on the symmetry and matter content.) We assume that the S field appears only in front of the gauge kinetic terms, and not in IR-brane superpotential terms, so that a large mass term for the Higgs doublets is not regenerated. (This may naturally occur in a UV theory in the absence of other gauge groups.) Note that, in the 4D description, this setup corresponds to the situation where the R-$SU(5)^2$ anomaly is canceled between the elementary-field and G-sector contributions.[5] In this setup, doublet-triplet splitting seems "natural," at least in the higher dimensional picture. Thus, while the theory with the R symmetry still seems to correspond to a particular choice of GUT breaking dynamics in the 4D description, we may say that the theory does not have the problem of doublet-triplet splitting.[6] After all, the "formulation" of the doublet-triplet splitting problem may have to be changed in

[5]We can show that this construction is not available in a 4D SUSY GUT theory where the GUT-breaking (Higgs) sector does not give tree-level contributions to the low energy anomalies. Assuming the MSSM matter content below the unification scale, with the $U(1)_R$ charges given by $V_{321}(0)$, $Q(1)$, $U(1)$, $D(1)$, $L(1)$, $E(1)$, $H_u(0)$ and $H_d(0)$, we find the low-energy $U(1)_R$-$SU(3)_C^2$, $U(1)_R$-$SU(2)_L^2$ and $U(1)_R$-$U(1)_Y^2$ anomalies to be 3, 1 and $-3/5$, respectively, which cannot be matched to high energy theories, where these anomalies arise as a $U(1)_R$-$SU(5)^2$ anomaly and are thus universal. (Here, the $SU(5)$ normalization is employed for the $U(1)_Y$ charges.) This implies that $U(1)_R$ should either be spontaneously broken, or there is explicit $SU(5)$-violating physics in the effective field theory. In our case, this conclusion can be avoided because the (dynamical) GUT-breaking sector carries the $U(1)_R$-$SU(5)^2$ anomaly, a part of which can be manifested as Green-Schwarz terms at low energies.

[6]An interesting feature of this class of theories is that the low energy theory contains an axion field S that couples to the QCD gauge fields with the decay constant of order the unification scale. This can be used to solve the strong CP problem [39], although the initial amplitude of this field in the early universe must be (accidentally) small to avoid the cosmological difficulty of overclosing the universe.

the large 't Hooft coupling regime, where physics is specified by the "hadronic" quantities, i.e. matter content, location, and boundary conditions in higher dimensions.[7]

In these respects, our framework offers many possible ways to address the problems of conventional 4D SUSY GUTs. For example, we can again consider a 5D $SU(5)$ gauge theory in the warped spacetime of Eq. (1), but then break the bulk $SU(5)$ by the VEV of a chiral superfield located on the IR brane, generated by an appropriate IR-brane superpotential. Then, if this superpotential does not have the problem of doublet-triplet splitting, e.g. by having the form of missing partner type models [4], then we may say that the problem has been solved. (As discussed before, it is better if the IR-brane superpotential is protected by a (discrete) symmetry; otherwise, it would correspond to "artificially" choosing the dynamics of the GUT breaking sector. In practice, this may be difficult, since we cannot use the non-universal Green-Schwarz terms on the IR brane because the GUT breaking there is due to the Higgs mechanism. We do not pursue this issue further here.) An advantage of this approach over conventional 4D model building is that we need not care about physics above the unification scale when engineering GUT breaking physics, i.e. the GUT-breaking Higgs content and superpotential. In the conventional 4D SUSY GUT framework, theories solving the doublet-triplet and/or dimension five proton decay problems often have too large matter content, leading to the problem of the unified gauge coupling hitting a Landau pole (well) below the gravitational/Planck scale. In our case, all (possibly large) multiplets located on the IR brane correspond to composite fields of the GUT breaking dynamics in the 4D description, and do not contribute to the running of the unified gauge coupling above the unification scale, $M_U \sim k'$ (see e.g. [40]). Potential complication of this sector may also not bother us, because it is the result of "dynamics" of the GUT breaking sector. We note that this makes the extension to $SO(10)$ unified theories trivial — we can break $SO(10)$ on the IR brane by arbitrary combinations of boundary condition and Higgs breakings with an arbitrary field content.

There are many applications of the ideas described above. For instance, we can apply it to product-group theories [7, 41, 42]. Let us once again consider a supersymmetric $SU(5)$ gauge theory in the 5D warped spacetime of Eq. (1). We then introduce an additional gauge group $SU(3) \times SU(2) \times U(1)$ on the IR brane, with the Higgs doublets charged under this IR-brane gauge group (and thus without being accompanied by any partner). Now, we can consider that our

[7]If we break the bulk $SU(5)$ gauge symmetry by boundary conditions at the UV brane, it leads to a theory which is interpreted as an $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory in the 4D description. The doublet-triplet splitting problem does not arise as the theory is not unified, and yet the successful unification of gauge couplings arises at the leading-log level in the limit that the tree-level gauge kinetic terms on the UV brane are small. This corresponds in the 4D description that the 321 gauge couplings at the unification scale are dominated by the asymptotically non-free contribution from a strong sector that has a global $SU(5)$ symmetry, of which the $SU(3) \times SU(2) \times U(1)$ subgroup is gauged and identified as the low-energy 321 gauge group. While this theory is somewhat outside the framework described in this paper, it is interesting on its own.

low-energy $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group is a diagonal subgroup of the bulk $SU(5)$ and the IR-brane $SU(3) \times SU(2) \times U(1)$. (This breaking can be caused by the VEV of an appropriate IR-brane localized field). Then, if the gauge couplings, $\tilde{g}_a$ of the original $SU(3)$, $SU(2)$ and $U(1)$ are large, $\tilde{g}_a \approx 4\pi$ $(a = 1, 2, 3)$, the low-energy MSSM gauge couplings are effectively unified at the scale where $SU(5) \times SU(3) \times SU(2) \times U(1) \to SU(3)_C \times SU(2)_L \times U(1)_Y$ occurs $\sim k'$, since the gauge couplings of $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$, g_a, are given by $1/g_a^2 = 1/g_5^2 + 1/\tilde{g}_a^2 \approx 1/g_5^2$ at that scale. Here, g_5 $(= O(1))$ is the coupling of the zero mode of the bulk $SU(5)$ gauge field. A problem of the corresponding scenario in 4D [42] is that, since the original $U(1)$ gauge coupling is strong at the unification scale, it hits the Landau pole immediately above that scale. There, it is also not clear why the three independent gauge couplings of $SU(3)$, $SU(2)$ and $U(1)$ become strong at a single scale, which must also coincide with the scale of diagonal breaking to avoid large threshold corrections. Our theory addresses all of these issues naturally — since the $SU(3)$, $SU(2)$ and $U(1)$ gauge fields are composite states of the GUT breaking dynamics, they all have strong couplings, $\tilde{g}_a \approx 4\pi$, at the scale where breaking to the diagonal subgroup occurs, and there is no issue of a Landau pole above this scale. A potentially large mass for the Higgs doublets can be avoided by introducing a (discrete) gauge symmetry with the anomalies canceled by the Green-Schwarz terms on the IR brane.[8] (To do all of these completely within the regime of effective field theory, the scale of diagonal breaking should be somewhat below the IR-brane cutoff, and the $SU(3)$, $SU(2)$ and $U(1)$ gauge couplings should be asymptotically non-free. These can be arranged with an appropriate introduction of massive fields on the IR brane. In the limit that the scale of diagonal breaking approaches the IR-brane cutoff, this theory is reduced to one of the theories discussed in the second paragraph of this subsection, where the Higgs doublets are located on the GUT-breaking IR brane.) Note that since the quarks and leptons are introduced in the bulk in representations of the bulk $SU(5)$, we are still considering a unified theory of quarks and leptons (although it is possible to introduce them on the IR brane in representations of $SU(3) \times SU(2) \times U(1)$). In particular, proton decay from unified gauge boson exchange still exists. The Yukawa couplings of matter to the Higgs fields arise through the IR-brane VEV, breaking $SU(5) \times SU(3) \times SU(2) \times U(1)$ down to $SU(3)_C \times SU(2)_L \times U(1)_Y$.

In most of the theories described above, the Higgs fields are localized to the IR brane, so that they are composite fields of the dynamical GUT breaking sector in the 4D description. (This need not be the case. One of the theories described in the second paragraph of this subsection contains Higgs fields that propagate in the bulk, with appropriate boundary conditions imposed at the IR brane. We can, however, always localize them to the IR brane by introducing appropriate hypermultiplet masses.) The observed hierarchies of quark and lepton masses and

[8] Another possibility for canceling anomalies is to add extra matter fields (in complete $SU(5)$ multiplets) that obtain masses from supersymmetry breaking [43]. We thank N. Maru for pointing out this work to us.

mixings can then always be explained by wavefunction overlaps between the matter and Higgs fields, by appropriately choosing the bulk hypermultiplet masses for the matter fields such that lighter generations are localized more towards the UV brane, as e.g. in Eq. (21). (The option of localizing lighter generations towards the IR brane is not available unless the Higgs fields are pseudo-Goldstone boson multiplets.) We find it very interesting that our framework of "holographic grand unification" accommodates many different ideas of solving the problems of conventional SUSY GUTs, developed mainly in the 4D context, with the automatic bonus of explaining the observed hierarchies of fermion masses and mixings through the wavefunction profiles of matter fields in the extra dimension.

5 Discussion and Conclusions

In this paper we have studied a framework in which grand unification is realized in truncated warped higher dimensional spacetime, where the UV and IR branes set the Planck and unification scales, respectively. In the 4D description, this corresponds to theories in which the the grand unified gauge symmetry is spontaneously broken by strong gauge dynamics having a large 't Hooft coupling, $\tilde{g}^2 \tilde{N}/16\pi^2 \gg 1$ (and a large number of "colors", $\tilde{N} \gg 1$). In this parameter region, an appropriate (weakly coupled) description of physics is obtained in higher dimensions, and physics above the unification scale is determined by higher dimensional field theories, e.g. by specifying the spacetime metric, gauge group, matter content, boundary conditions, and Lagrangian parameters. This allows us to control certain dynamical properties of the GUT breaking sector in the regime where effective field theory applies. For example, we can make the size of threshold corrections small by making the symmetry breaking VEV on the IR brane (slightly) smaller than its naive value. Moreover, the framework allows us to straightforwardly adopt intuitions and mechanisms arising from the higher dimensional picture. In particular, we can explain the observed hierarchies in quark and lepton masses and mixings in terms of the wavefunction profiles of matter fields in higher dimensions. The generated hierarchies are naturally of the right size, of order $M_U/M_* \simeq 1/20$.

We have presented several realistic models within this framework. In one model, on which we have focused the most, the lightness of the Higgs doublets is explained by the pseudo-Goldstone mechanism. The strong gauge dynamics sector possesses a global $SU(6)$ symmetry as a "flavor" symmetry, of which the $SU(5)$ ($\times U(1)$) subgroup is gauged and identified as the unified gauge group. When the global symmetry is broken dynamically to $SU(4) \times SU(2) \times U(1)$, the unified gauge symmetry is broken to the standard model gauge group, and the two MSSM Higgs doublets arise as massless pseudo-Goldstone supermultiplets. In our framework, this is realized by postulating a bulk $SU(6)$ gauge symmetry, broken to $SU(5) \times U(1)$ and $SU(4) \times SU(2) \times U(1)$

on the UV and IR branes, respectively. One of the difficulties in implementing this mechanism in the conventional 4D framework is to find a way to suppress effects of explicit breaking in the potential generating the spontaneous $SU(6)$ breaking, since such effects would reintroduce an unacceptably large mass for the Higgs doublets. In our case, these effects are (exponentially) suppressed by a large mass dimension for the operator generating the spontaneous $SU(6)$ breaking. Such an assumption is easy to implement in higher dimensions – simply assume that the Higgs field breaking $SU(6)$ is localized to the IR brane. This provides another example of the "controllability" of strong gauge dynamics in the large 't Hooft coupling regime.

We have also demonstrated that many ideas for solving the problems of conventional 4D SUSY GUTs can be naturally implemented on the IR brane. We have presented several realistic models of this kind, for example, ones based on missing partner type or product group type scenarios. These models have the interesting feature that the GUT scale physics on the IR brane does not affect physics at higher energies, since the relevant physics arises as a result of the strong GUT breaking dynamics (as composite states) in the 4D description. For example, large GUT multiplets, often needed to solve the problems of SUSY GUTs, do not contribute to the evolution of the unified gauge coupling at higher energies, and gauge fields having very large gauge couplings can naturally arise at the GUT scale without having the problem of a Landau pole. These features open up new possibilities for GUT model building.

One can view the "success" of the present framework in several different ways. For one who is interested in addressing the phenomenology of unified theories, such as gauge coupling unification and proton decay, models in our framework can be used to give predictions of observable quantities. For example, we can explore relations between the branching ratios of proton decay and matter configurations in the extra dimension, as in the case of unified theories in flat space [44]. Models of fermion masses and mixings, as well as models of supersymmetry breaking, can also be developed within the framework.[9] On the other hand, one may be interested in exploring possible "UV completions" of models formulated in warped spacetime. It is possible, after all, that there may be some nontrivial consistency conditions in higher dimensional field theories,

[9]For example, we can take one of the supersymmetry breaking models in [47], with the boundary conditions at the UV brane changed to be trivial, and glue that spacetime (the 5D warped spacetime with the scales at the UV and IR branes taken to be the Planck and TeV scales, respectively) to one of our holographic warped GUT spacetimes discussed in section 4, at the UV branes of both spacetimes. (The consistency of such constructions in effective field theory has been discussed recently in [48].) In the 4D description, this corresponds to the situation where both the unified gauge symmetry and supersymmetry are broken by strong gauge dynamics, at the unification scale and the TeV scale, respectively. In practice, this system is analyzed most efficiently by first integrating out the GUT scale physics. Then the low energy effective theory is simply reduced to one of the models in [47], but now we have an understanding of the hierarchical structure of the Yukawa couplings, located on the UV brane of the effective theory. While this effective field theory may be at the border of the weak and strong coupling regimes in 5D, it may still reproduce gross features of physical quantities, e.g. the superparticle spectrum, as is the case in higher dimensional formulations of QCD.

which are difficult (though not impossible) to catch in effective theory, and one way of ensuring the consistency of such theories is to "derive" them from complete UV theories. Such "UV completions" may be achieved, for example, by embedding models into string theory, identifying a "dual" 4D theory, or by finding a 4D theory whose infrared fixed point has similar features as the original models in warped space [45]. From this perspective, our framework offers a guide on which models "UV theorists" should aim to reproduce; for example, string theorists may want to reproduce unified theories in 5D warped spacetime, with the unified gauge symmetry broken at an IR throat, rather than 4D unified theories directly from compactification.

We finally comment on the possibility that the unified gauge symmetry is broken by strong gauge dynamics whose 't Hooft coupling is large but not extremely large, i.e. $\tilde{g}^2\tilde{N}/16\pi^2 \sim 1$. In this case, the picture based on higher dimensional spacetime is not fully justified, but even then some properties of theories, especially properties associated with the IR brane physics (GUT breaking dynamics), may be effectively described by our higher dimensional warped unified theories. In fact, such an approach had a certain level of successes in describing physics of lowest-lying excitations in QCD [46]. In this sense, our framework may have a larger applicability than what is naively expected.

In summary, we have presented a framework in which dynamical GUT breaking models are realized in a regime that has a weakly coupled "dual" picture. Grand unified theories are realized in warped higher dimensional spacetime, with the UV and IR spacetime cutoffs providing the Planck and the unification scales, respectively. Several types of realistic models are discussed, with interesting implications for quark and lepton masses and mixings. It would be interesting to study further implications of these models, such as those on proton decay, precise gauge coupling unification, supersymmetry breaking, and flavor physics.

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